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The beta spectrum of the fission products from ^{252}Cf and the associated inner bremsstrahlung radiation

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Abstract. Beta ray intensity against energy in the decay of ^{252}Cf has been calculated as a function of time and it is predicted that 7·0 beta rays are emitted per fission. The inner bremsstrahlung radiation from all decaying fragments is shown to be equivalent, in the first 40·96 s after fission, to the radiation from a black body which cools from $1{\cdot}5\times10^{10}$ to $1{\cdot}2\times10^{10}$ K.

1. Introduction

This is a calculation of the beta spectrum and associated inner bremsstrahlung and recoil radiation from a single spontaneous fission of californium 252.

This paper does not address itself to the purported difficulties that may exist in experimentally identifying this spectrum in the presence of fission related gamma rays. Windows do exist between these characteristic gamma rays through which the inner bremsstrahlung could be observed with proper instrumentation. These results may be of some significance to astrophysicists since the calculated temperature of the black body radiation by this process falls between that measured for pulsars and that calculated for a black hole in comparable parts of the spectrum.

The computer program and input data used have evolved over many years and have given results in the case of ^{235}U fissioned by thermal neutrons which are in excellent agreement with the doctoral thesis experimental results of Kutcher (1965), Tsoulfanidis (1968) and Wyman (1970).

2. Computation

The IBM-360/95 FORTRAN program for this model and method of computation are not reproduced here; however, we will provide this, as well as input data, on request. Basic concepts are briefly outlined.

2.1. Method

The disintegration rate $\beta_j(t)$ for the jth fission fragment of a chain was generated by solving the following differential equation for the decay chain:

$$\frac{\mathrm{d}A_j(t)}{\mathrm{d}t} + \lambda_j A_j(t) = \lambda_{j-1} A_{j-1}(t) \tag{1}$$

where

$$\beta_j(t) = \lambda_j A_j(t)$$

$A_j(t)$ is the amount of the jth isotope at time t, with decay constant λ_j, in terms of all previous members of a chain and their decay constants. $\beta_j(t)$ was then multiplied by a function

$$\rho_j(E)\,\mathrm{d}E = \frac{E(E^2-1)^{1/2}(E_0-E)^2 F(Z+1, E)\{(E^2-1)+(E_0-E)^2\}^\alpha\,\mathrm{d}E}{\int_1^{E_0} E(E^2-1)^{1/2}(E_0-E)^2 F(Z+1, E)\{(E^2-1)+(E_0-E)^2\}^\alpha\,\mathrm{d}E} \tag{2}$$

which is used as the probability that a beta ray emitted from the jth isotope has an energy between E and $E+\mathrm{d}E$. In equation (2), E is the beta energy in rest mass (m_0c^2) units, E_0 the beta end point energy in m_0c^2 units, $F(Z+1, E)$ the energy correction term due to the Coulomb distortion of an electron wave amplitude in the Coulomb field of the nucleus, and α is the degree of forbiddenness of the transition. $Z+1$ is used for computer book-keeping and refers to the daughter nucleus.

In our study $\alpha = 0$ covers all allowed and once forbidden transitions having allowed shapes. Other once or twice forbidden transitions are covered by the use of $\alpha = 1$. Strictly speaking $\alpha = 1$ refers to the unique first forbidden decay only. For computation ease since only 2 or 3 of the greater than 400 products have second forbidden transitions or higher, and these have very low initial abundances, the use of $\alpha = 1$ is justified. The overall answer is not effected.

The product $\beta_j(t)\rho_j(E)\,\mathrm{d}E$ gives the number of beta rays at a given energy and time from the jth isotope, in the energy range between E and $E+\mathrm{d}E$.

This product is then summed over all known or postulated members j of a specific fission chain with mass number A, then summed over A ranging from 72 to 162 so that

$$\beta(E, t) = \sum_A \sum_j \beta_j(t)\rho_j(E). \tag{3}$$

Beta decays of a given isotope having several different beta branches are handled by setting up the input data to recycle through all the different end point energies needed in the calculation.

2.2. *Input data*

Equation (3) requires input data of two general categories as follows: (i) the initial fission abundance $A_j(0)$ of each isotope, by charge distribution Z within the chain A, and λ_j from which $\beta_j(t)$ is obtained; (ii) the beta ray end point energy E_0, and the degree of forbiddenness of each transition, from which $\rho_j(E)$ is calculated.

A complete compilation of all the data used in our calculations is available on IBM punched cards.

The total chain yields for ^{252}Cf used were those given by Schmitt *et al* (1965). In regions of incomplete data, yields were taken from figure 7 of Schmitt *et al* (1965). The total fission yield was normalized to 200%.

The fractional chain yields were determined for each value of Z in the mass chain A in the region of the most probable charge in fission Z_p for that mass number. The values for Z_p for thermal neutron fission of ^{235}U were taken from Coryell and Sugarman (1951). The change in Z_p for the ^{252}Cf case of spontaneous fission, relative to that obtained in the thermal neutron fission of ^{235}U, was found through the use of Coryell's prescription

$$\Delta Z_p(A) = \tfrac{1}{2}(Z_c-92)-0{\cdot}21(A_c-236)+0{\cdot}023(E^*-6{\cdot}5) \qquad (4)$$

where $\Delta Z_p(A) = Z_{p*}(A) - Z_p(A)$ (standard), $Z_{p*}(A)$ = the value sought for ^{252}Cf spontaneous fission, $Z_p(A)$ (standard) = the value of Z_p of mass chain A for thermal neutron fission of ^{235}U. Z_c = the charge 98, A_c = the mass 252, E^* = excitation energy (0 meV) of ^{252}Cf. Beta end point energies E_0 and decay rates are the latest experimental of theoretically predicted results.

3. Inner bremsstrahlung and recoil radiation

The calculations up to this point give the number of beta rays per second per 10 000 eV energy interval per fission as a function of time after fission. Figure 1 lists the calculated results.

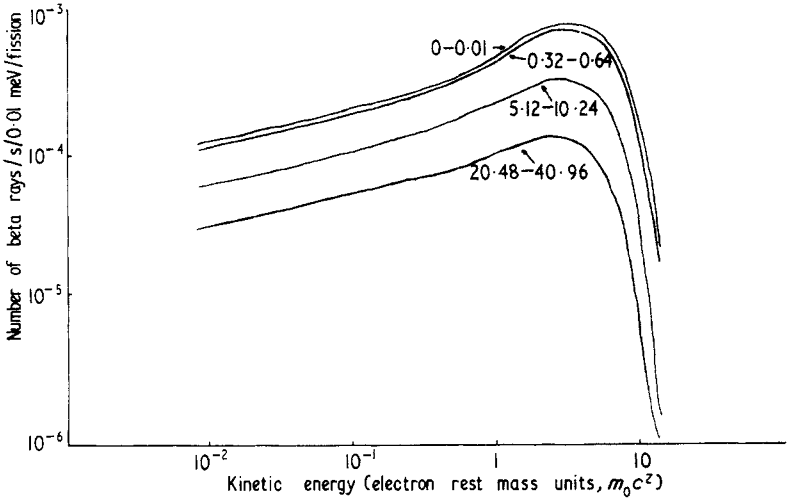

Figure 1. ^{252}Cf fission fragment beta spectrum. The numbers refer to times after fission in seconds.

Table 1 lists the time dependence of the intensity of beta rays (all energies) and predicts a total of 7·0 beta rays emitted per spontaneous fission. This result is yet to be experimentally verified. However, the calculational capability of this model has given exceptionally good values in the case of ^{235}U, namely 5·9 beta rays per fission.

Armed with a computer program capable of following each of over 400 fission fragments through its detailed beta intensity, energy and time relationships, it is now possible to calculate the inner bremsstrahlung radiation intensity against time.

Our program incorporates the theoretical method, outlined by Jackson (1962). We have extended the beta energy range considerably below the 630 eV usually given in the National Bureau of Standards table of energy correction terms due to the Coulomb

Table 1. Time rate of beta emission from fission fragments of ^{252}Cf

Time interval after fission (s)	Number of beta rays emitted (s^{-1})
0·0	0·29149050E 00
0·99999979E-02	0·29033226E 00
0·19999996E-01	0·28918767E 00
0·39999992E-01	0·28694147E 00
0·79999983E-01	0·28260428E 00
0·15999997E+00	0·27449256E 00
0·31999993E 00	0·26011515E 00
0·63999987E 00	0·23653942E 00
0·12799997E 01	0·20131183E 00
0·25599995E 01	0·15490961E 00
0·51199989E 01	0·10608423E 00
0·10239998E 02	0·66792428E-01
0·20479996E 02	0·38554832E-01
0·40959991E 02	0·19016944E-01
0·81919983E 02	0·81994124E-02
0·16383997E 03	0·36440671E-02
0·32767993E 03	0·16512882E-02
0·65535986E 03	0·75618808E-03
0·13107197E 04	0·36173617E-03
0·26214395E 04	0·16731628E-03
0·52428789E 04	0·63785323E-04
0·10485758E 05	0·23763045E-04
0·20971516E 05	0·10877071E-04
0·41943031E 05	0·56539320E-05
0·83886063E 05	0·28859140E-05
0·16777213E 06	0·13600575E-05
0·33554425E 06	0·59443602E-06
0·67108850E 06	0·26379718E-06
0·13421770E 07	0·12510901E-06
0·26843540E 07	0·55622444E-07
0·53687080E 07	0·20982071E-07
0·10737416E 08	0·86990006E-08
0·21474832E 08	0·43465676E-08
0·42949664E 08	0·23108437E-08
0·85899328E 08	0·91361629E-09
0·17179866E 09	0·21865547E-09
0·34359731E 09	0·60228544E-10
0·68719462E 09	0·31651431E-10
0·13743892E 10	0·19237889E-10
0·27487785E 10	0·82210306E-11

Total number of beta rays 7·0

distortion of an electron wave amplitude in the Coulomb field of the nucleus. For a few additional minutes of computer time we have also calculated the electromagnetic spectrum of the recoil radiation due to the momentum change of the ionized fragments. This classical recoil radiation (Jackson 1962) is in coincidence with the inner bremsstrahlung due to beta emission. The intensity of the recoil radiation depends upon the energy of the recoiling ion, and the latter is given, for example by Wu and Moszkowski

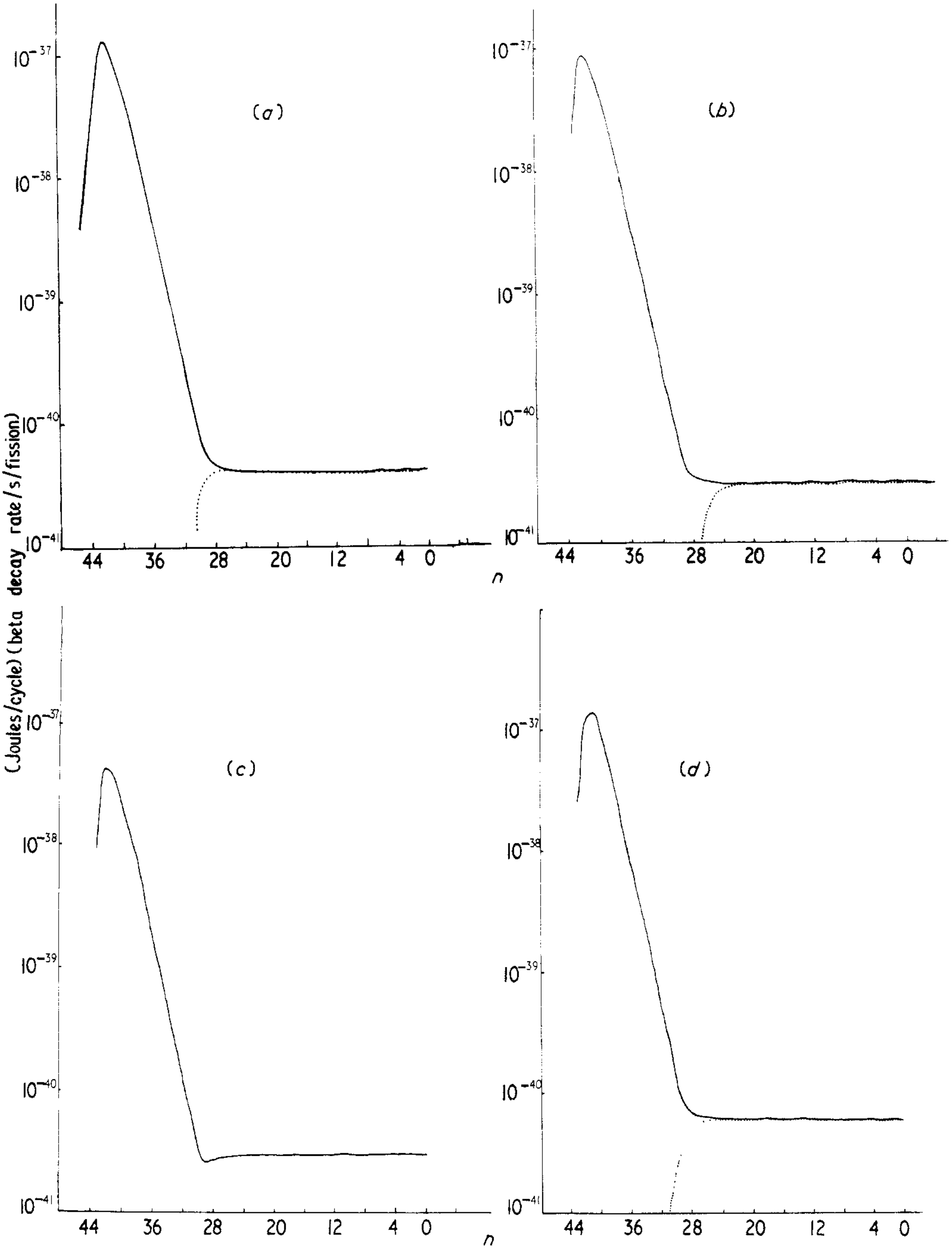

Figure 2. Inner bremsstrahlung and recoil radiation from the beta spectrum of fission fragments from ^{252}Cf. (*a*) Time after fission 0–0·01 s; HF black body equivalent temperature (BBET) $1{\cdot}5 \times 10^{10}$ K; LF BBET 4×10^{8} K; dotted line, contribution from recoil radiation alone. (*b*) Time after fission 1·28–2·56 s; HF BBET $1{\cdot}4 \times 10^{10}$ K; LF BBET $3{\cdot}9 \times 10^{8}$ K; dotted line, contribution from recoil radiation alone. (*c*) Time after fission 5·12–10·24 s; HF BBET $1{\cdot}3 \times 10^{10}$ K. (*d*) Time after fission 20·48–40·96 s; HF BBET $1{\cdot}2 \times 10^{10}$ K; LF BBET $3{\cdot}5 \times 10^{8}$ K; dotted line, contribution from recoil radiation alone. Frequency expressed in terms of *n*, is given by $2^{n+1} \times 10^{8}$ Hz.

(1966) as

$$E_r = \frac{533\times10^{-6}}{A}[E(E+1{\cdot}02)+(E_0-E)^2+2(E_0-E)\{E(E+1{\cdot}02)\}^{1/2}\cos\theta] \tag{5}$$

where E_r is the kinetic energy of the recoil ion (in MeV), A is the mass number of the ion, E and E_0 are the energy and maximum energy of the beta particle and θ is the angle between the emitted electron and neutrino. Our calculations use centre of mass co-ordinates and the program performs a Monte Carlo calculation on the direction of beta emission. Subsequent modifications to include a time dependency in coincidence with beta decay were included in the computer program.

The individual and combined results are given in figure 2(*a–d*). In order to compare these radiations with astrophysical emitters our program also compares the high frequency intensities and low frequency intensities to equivalent black body radiators. These comparisons are done by computerized curve fitting routines and show (see figures 2(*a–d*)) that in the ultraviolet, x ray and soft gamma ray regions the inner bremsstrahlung radiation appears as a black body which, as a function of time, cools from $1{\cdot}5\times10^{10}$ to $1{\cdot}2\times10^{10}$ K. In the microwave, infrared and visible regions the combined inner bremsstrahlung and recoil appear as a black body with temperature which cools in the first 40·96 s from 4×10^8 to $3{\cdot}5\times10^8$ K. Almost the entire low frequency intensity is due to recoil radiation. In the microwave to infrared region the shape of the curve is strongly dependent upon the neutrino's energy sharing with the beta ray.

It is interesting to note that the black body temperature ranges, equivalent to the inner bremsstrahlung and recoil radiation from beta transitions in nuclear masses in the fission fragment lie at the lower limit of black hole compression (Doroshkevich *et al* 1970) and the upper limit of pulsars.

Acknowledgments

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References

Coryell C D and Sugarman N 1951 *National Nuclear Energy Series Section* 4 vol 9 (New York: McGraw-Hill)
Doroshkevich A G, Zel'dovich Ya B and Novikov I D 1965 *Zh. eksp. teor. Fiz.* **49** 170 (1966 *Sov. Phys.-JETP* **22** 122)
Jackson J D 1962 *Classical Electrodynamics* (New York: Wiley) pp 526–8
Kutcher J W 1965 *PhD Thesis* University of Illinois
Schmitt H W, Kiker W E and Williams C W 1965 *Phys. Rev.* **137** B837–47
Tsoulfanidis N 1968 *PhD Thesis* University of Illinois
Wu C S and Moszkowski S A 1966 *Beta Decay* (New York: Interscience) pp 106–10
Wyman M E 1970 *PhD Thesis* University of Illinois